Controlled Stratification for Determining Extreme Quantiles in Electromagnetic Compatibility Risk Analysis

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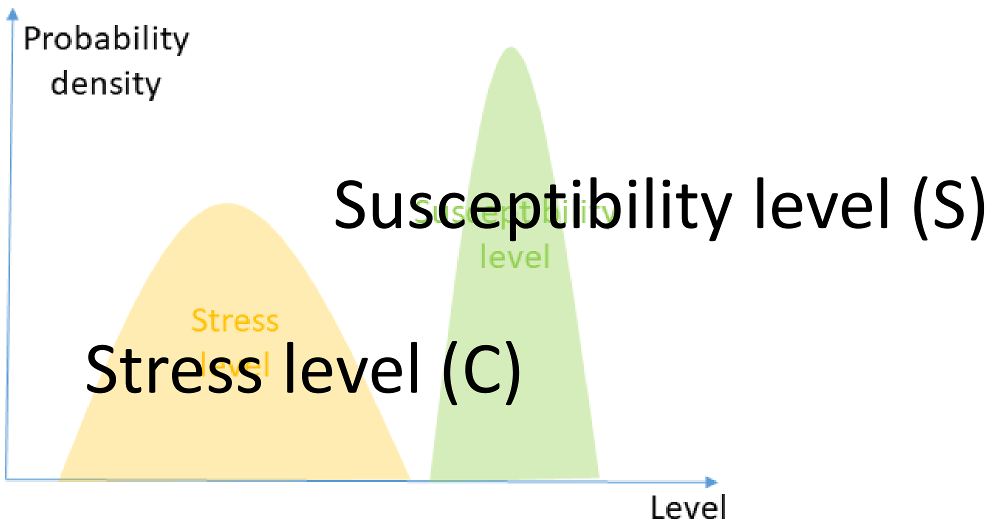
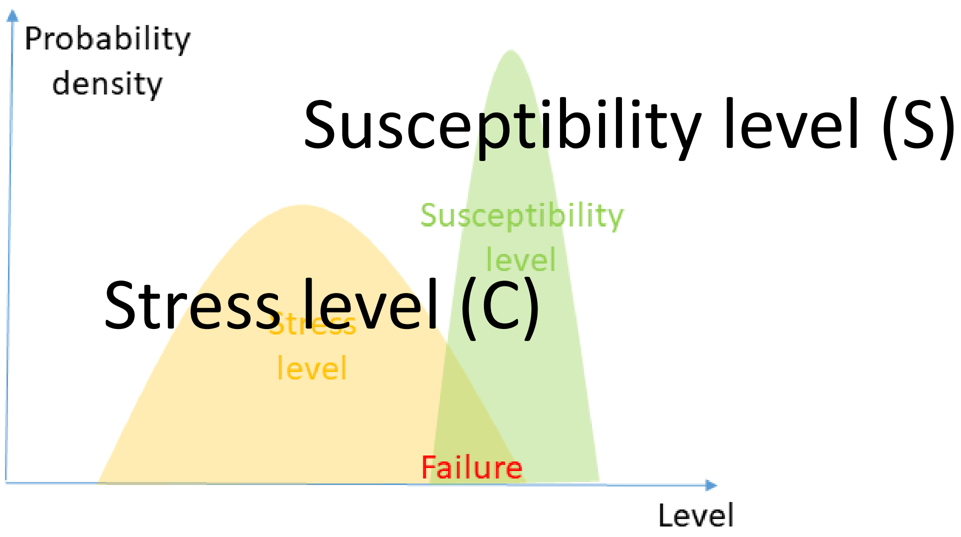
A general framework: EMC risk analysis



(a) A risk-free configuration



(b) A possible risk configuration

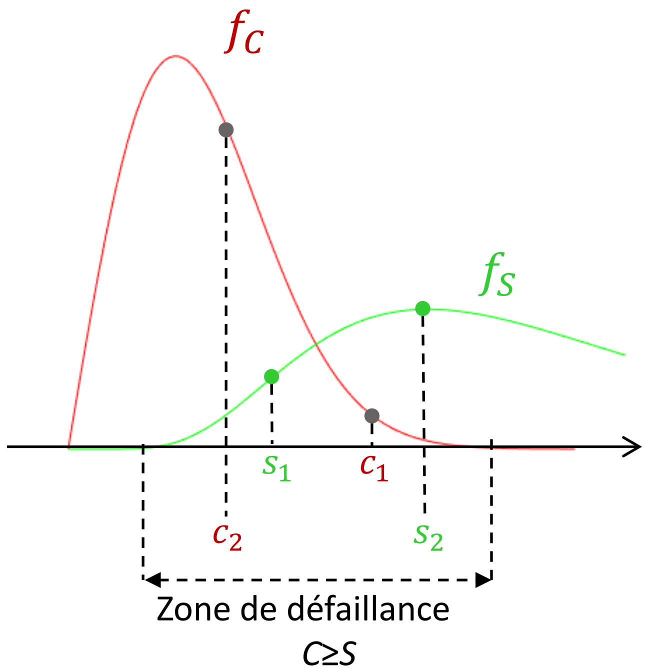
A general framework: EMC risk analysis

- Probability of failure

$$\mathbb{P}\{C \geq S\} = \int_{-\infty}^{\infty} \left(1 - F_C(x)\right) f_S(x)\, dx$$

Stress

Susceptibility



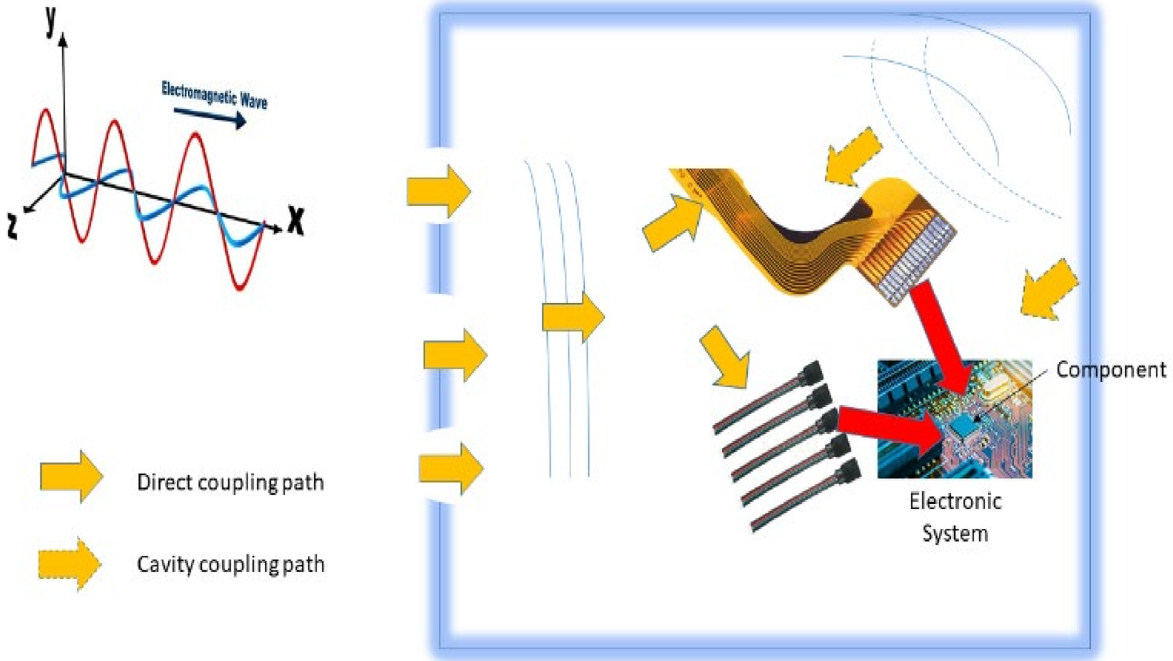
Distributions are not needed but part of them under the following hypotheses:

- If they strongly overlap, $\mathbb{P}\{C \geq S\}$ is significant ; High risk
- If distributions are known to be far apart : $\mathbb{P}\{C \geq S\}$ --> 0 ; Negligible risk
- Otherwise, $\mathbb{P}\{C \geq S\}$ to be estimated

EMC scenario (radiatd susceptibility)

Discussion about one example

Stress level: numerical modeling



Susceptibility level ?
A parametric model ?
A statistical model from a masured sample?

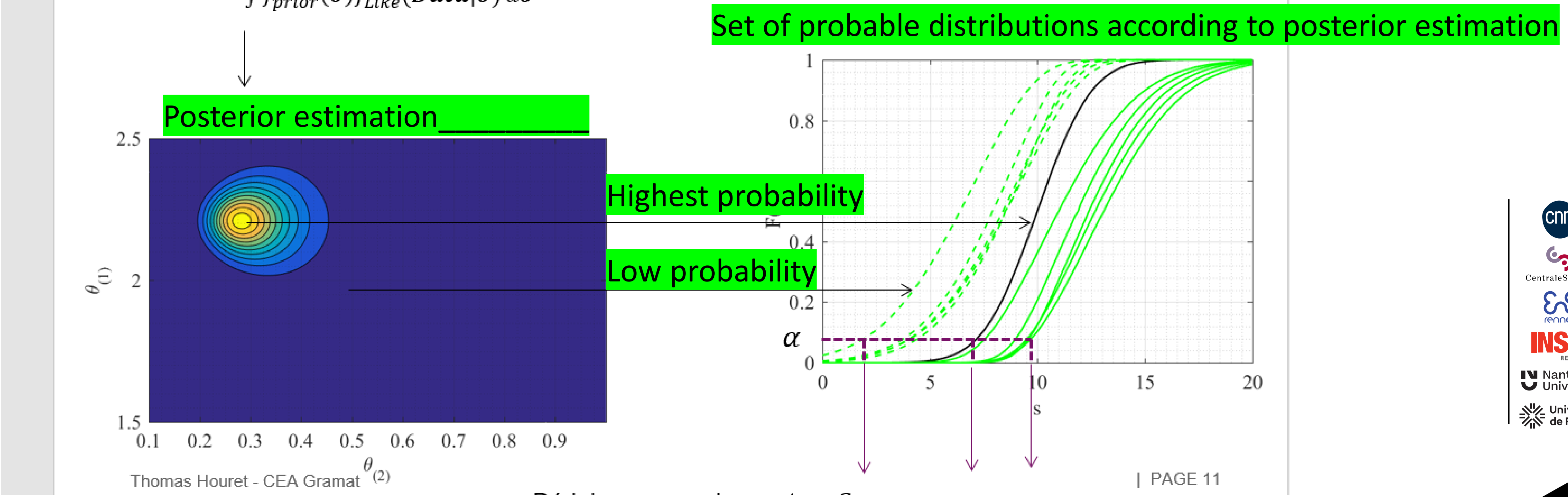
EMC scenario (radiated susceptibility)

Susceptibility level ?

A statistical model from a measured sample (statistical inference: Bayes)

T. Houret et. al., "Estimating the Probability Density Function of the Electromagnetic Susceptibility from a Small Sample of Equipment,“ Progress in Electromagnetics Research B, Vol. 83, pp. 93-109, Feb. 2019

$$f_{post}(\boldsymbol{\theta}|\boldsymbol{Data}) = \frac{f_{prior}(\boldsymbol{\theta}) f_{Like}(\boldsymbol{Data}|\boldsymbol{\theta})}{\int f_{prior}(\boldsymbol{\theta}) f_{Like}(\boldsymbol{Data}|\boldsymbol{\theta})\, d\boldsymbol{\theta}}$$

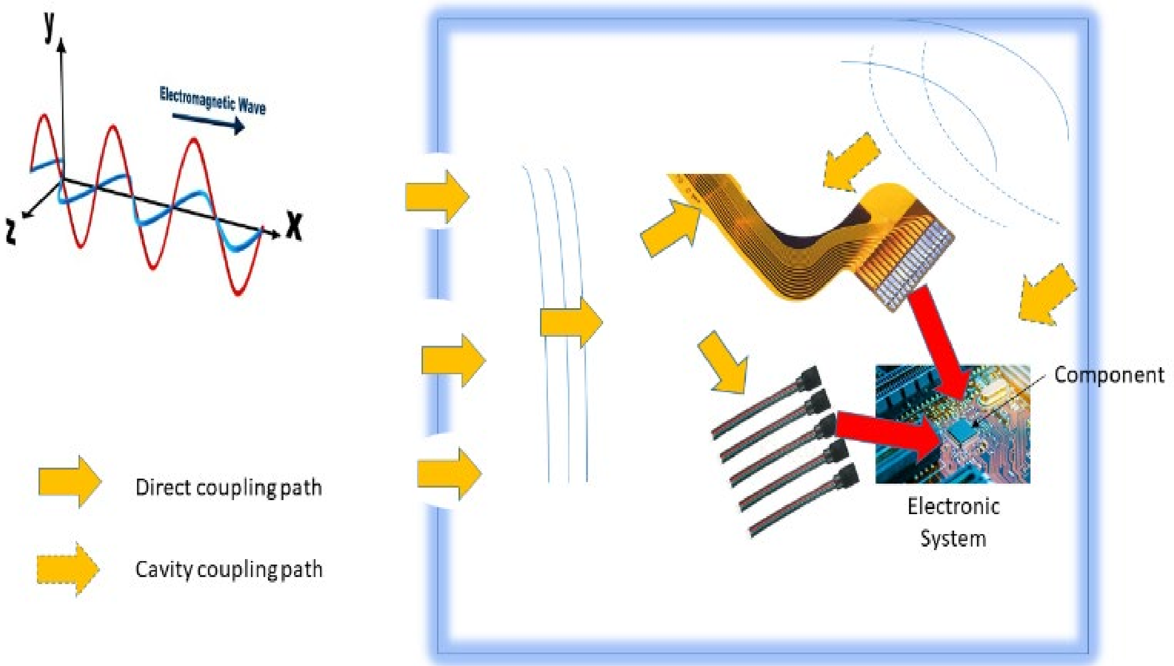


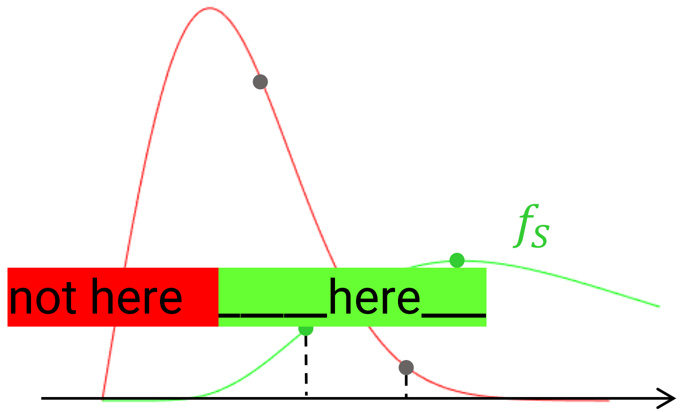
EMC scenario (radiated susceptibility)

What about the stress level ?

Numerical and parametric modeling is appropriate

But smart sampling (here / not here) is even more





We need a statistical trick here!

Introduction to controlled stratification

- ✓ We have a trustable numerical model M:

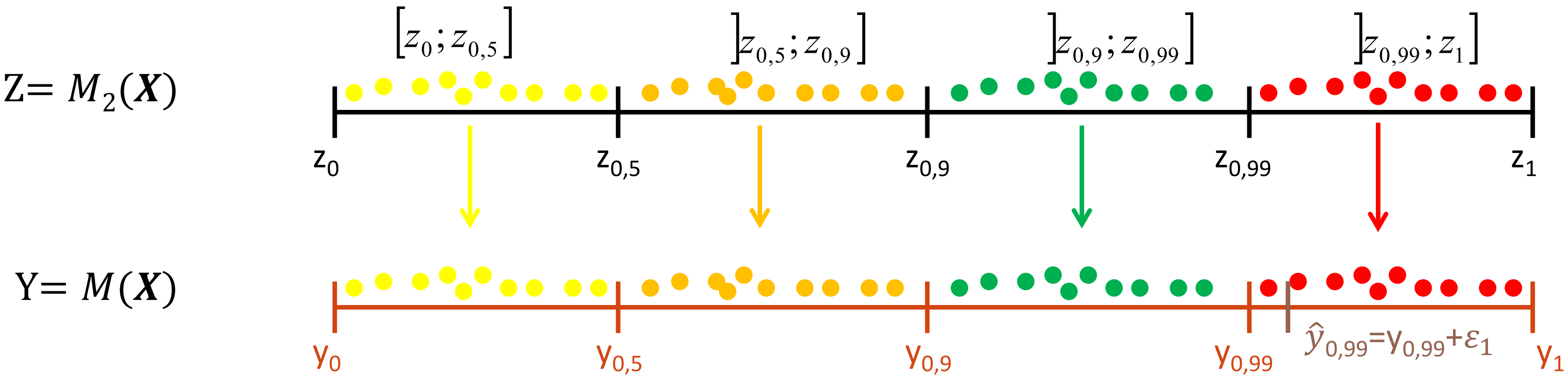
$$\boldsymbol{y} = M(\boldsymbol{X})$$

- ✓ Preferably, we aimt at sampling particular subspaces of $\boldsymbol{X}$ with the highest $\boldsymbol{y}$ output

- ✓ What do we need ?
  - ✓ At least some runs to learn about an estimation of, say, some quantile of $\boldsymbol{y}$
  - ✓ And … HELP!

Introduction to controlled stratification (CS)

CS : using a auxiliary model (M_2) to estimate the output extreme quantile of the original model (M)



The target:

$$\hat{Y}(\alpha) = \inf\left\{y, \hat{F}(y) > \alpha\right\}$$

M_2 is a auxiliary model with two specific properties.

- It provides similar trends regarding the output ranking (likely the same bins between quantiles)
- It is run for almost free (a few seconds or minutes)

How to find it ?

- Lucky you, if you find it!

Introduction to controlled stratification (CS)

M_2 provides a smart sampling of X for the estimation of an extreme quantile of Y through M

Possible Z candidates ?

- ✓ M_2 may be a simplified version of M preserving physics (or most part of it)
  - ✓ Fullwave simulation approximated by some circuit models
  - ✓ Fullwave simulation with coarse meshes (low-fidelity simulations)
- ✓ M_2 could be any surrogate model

How do we know about M_2 being a good candidate or not ?

- ✓ Metric based on the correlation of the ouput ranking according to predefined strata
- ✓ The higher the correlation the faster the estimator converges
- ✓ A gamble: to be checked afterwards!

Introduction to controlled stratification

$$\hat{\rho}_I = \left. \frac{\sum_{j=1}^{n} \left(1_{Z_j \le z_\alpha} - \hat{G}_n(z_\alpha)\right)\left(1_{Y_j \le y_\alpha} - \hat{F}_{EE}(y_\alpha)\right)}{\sqrt{\sum_{j=1}^{n} \left(1_{Z_j \le z_\alpha} - \hat{G}_n(z_\alpha)\right)^2} \sqrt{\sum_{j=1}^{n} \left(1_{Y_j \le y_\alpha} - \hat{F}_{EE}(y_\alpha)\right)^2}} \right|_{y_\alpha = \hat{Y}_{CV}(\alpha)}$$

where $\hat{F}_{EE}(y_\alpha) = \frac{1}{n}\sum_{i=1}^{n} 1_{Y_i \le y_\alpha}$ and $\hat{G}_n(z_\alpha) = \frac{1}{n}\sum_{i=1}^{n} 1_{Z_i \le z_\alpha}$

1) If $Z_j \le z_\alpha$ and $Y_j \le y_\alpha$: (1-0.99)*(1-0.99)=(0.01)*(0.01)=$1*10^{-4}$
2) If $Z_j > z_\alpha$ and $Y_j \le y_\alpha$: (1-0.99)*(0-0.99)=(0.01)*(-0.99)=$-9.9*10^{-3}$
3) If $Z_j \le z_\alpha$ and $Y_j > y_\alpha$: (0-0.99)*(1-0.99)=(-0.99)*(0.01)=$-9.9*10^{-3}$
4) If $Z_j > z_\alpha$ and $Y_j > y_\alpha$: (0-0.99)*(0-0.99)=(-0.99)*(-0.99)=0.9801

Measures the correlation of the models at the tail of their distributions

Introduction to controlled stratification

The cdf of Y can be defined from a conditional probability involving Z:

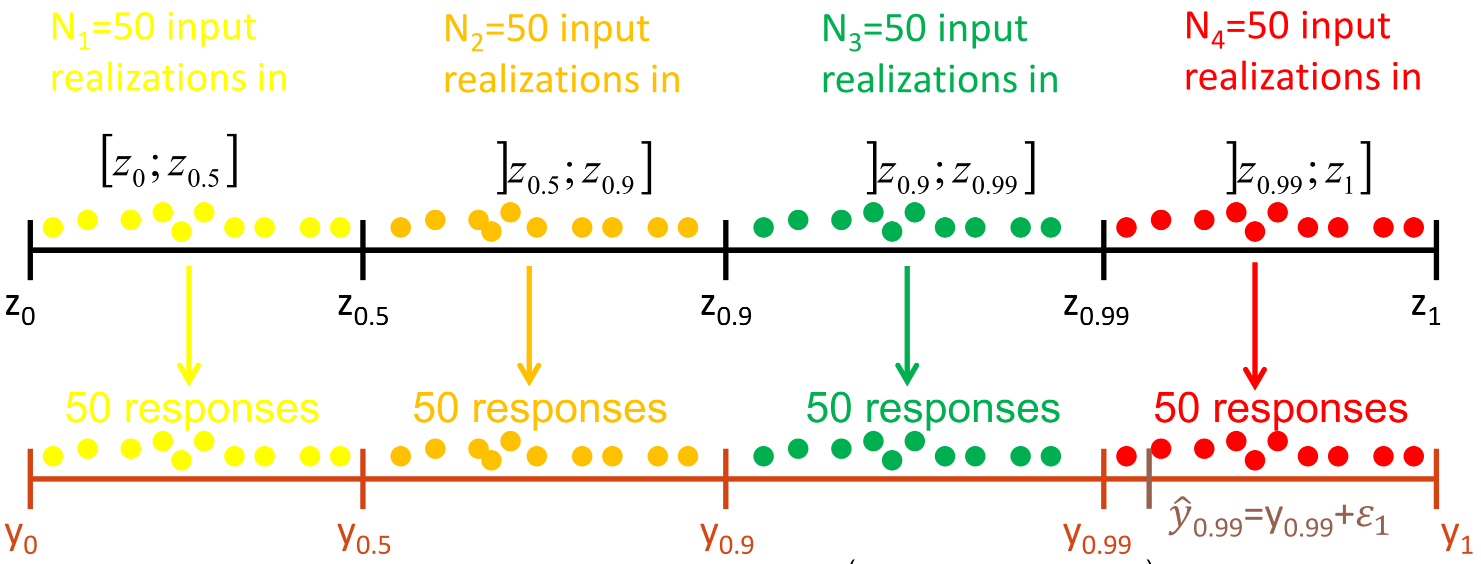
$$F(y) = \sum_{j=1}^{m} P_j(y)\,(\alpha_j - \alpha_{j-1}) \qquad P_j(y) = P(Y \le y / Z \in]z_{\alpha_{j-1}}, z_{\alpha_j}])$$

where $P_j(y)$ is the conditional probability

If $\widehat{\rho}_{\mathrm{I}} = 0$ then $P_j(y) = P(Y \le y)$

On the contrary, if $\widehat{\rho}_{\mathrm{I}}$ is significant, the stratification allows populating the tail of Y distribution

Introduction to controlled stratification



Quantile estimation: $\hat{Y}_{CS}(0.99) = \inf\{y, \hat{F}_{CS}(y) > 0.99\}$

where $\hat{F}_{CS}(y) = 0.5 * \hat{P}_1(y) + 0.4 * \hat{P}_2(y) + 0.09 * \hat{P}_3(y) + 0.01 * \hat{P}_4(y)$

$$\hat{P}_j(y) = \frac{1}{N_j} \sum_{i=1}^{N_j} 1_{Y_i^{(j)} \leq y / Z \in]z_{\alpha_{j-1}} ; z_{\alpha_j}]}$$

$\widehat{\rho_i} = 1$

$$F_{CS}(\widehat{y_{0.99}}) = 0.5 \times (50/50) + 0.4 \times (50/50) + 0.09 \times (50/50) + 0.01 \times (1/50) = 0.9902$$

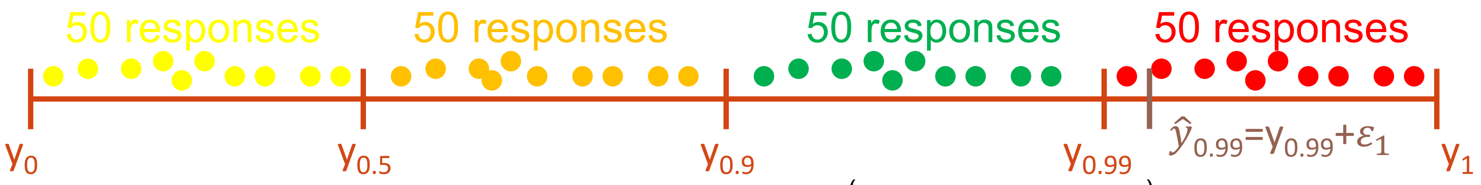
$\widehat{\rho_i} = 0$

$$F_{CS}(\widehat{y_{0.99}}) = ?$$

$$\sqrt{n}(\hat{Y}_{EE}(\alpha) - y_\alpha) \xrightarrow[n \to +\infty]{} \mathcal{N}(0, \sigma_{EE}^2),$$

$$\sigma_{EE}^2 = \frac{\alpha(1-\alpha)}{p^2(y_\alpha)}.$$

Controlled Stratification: what are the gains?

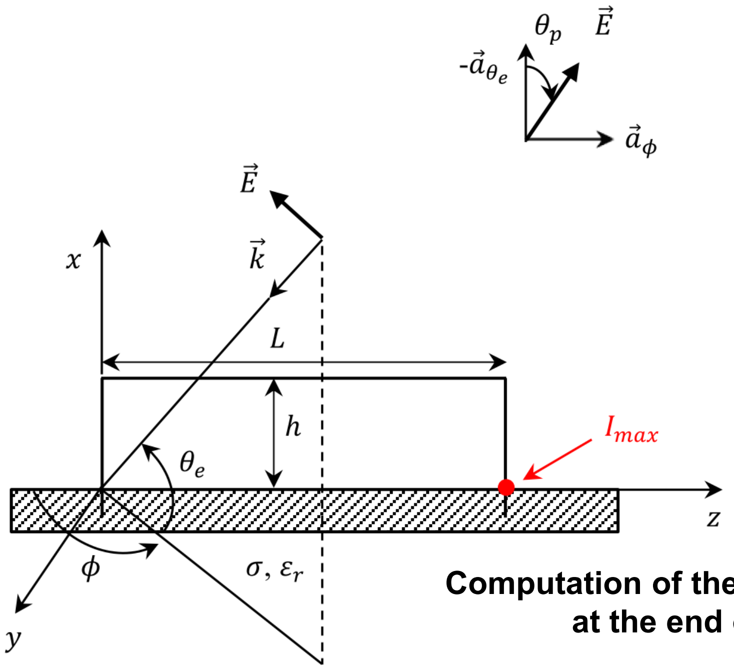


Quantile estimation: $\Rightarrow$ $\hat{Y}_{CS}(0.99) = \inf\left\{y, \hat{F}_{CS}(y) > 0.99\right\}$

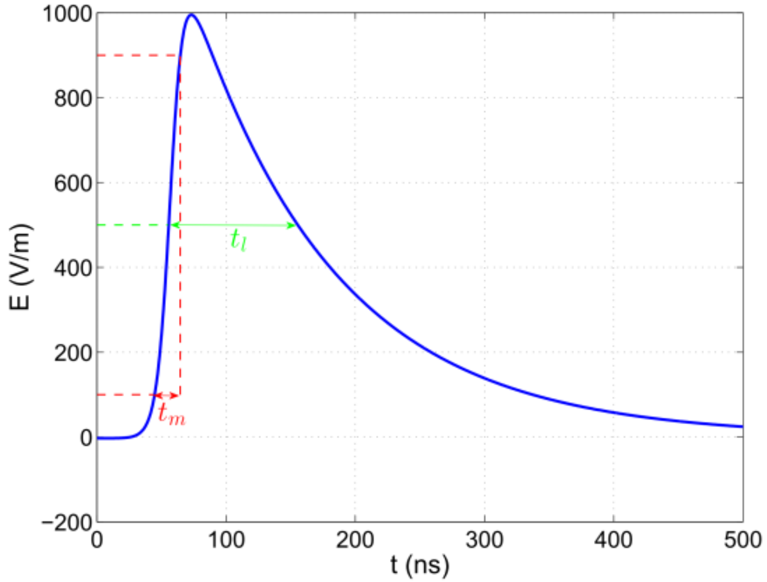
Variance reduction of the quantile estimation: $1-\rho_{\mathrm{I}}^2$

A significant sample of extreme values (POT) events -> extreme value distribution

Application to a coupling of a plane wave with an aerial electrical cable (NEMP)



Computation of the current I_{max}=max |I| at the end of the cable



- Set for the computation
  - Incident plane wave with a normalized peak electric field strength: $\vec{E}$=1000 V/m and a rise time tm=20 ns → Parameters
  - tl ∈ [50; 500 ns] is the decay time of the incident wave
  - θp ∈ [0; 90°] is the polarisation angle of the incident wave
  - θe ∈ [0; 90°] is the elevation angle of the incident wave
  - ϕ ∈ [0; 359°] is the azimuth angle of the incident wave
  - cable : L ∈ [100; 500 m], h ∈ [6; 12 m], d ∈ [1; 20 cm]
  - Ground conductivity $\sigma=10^{-3}$ S/m, and the relative permittivity ε_r = 1 → Parameters

Uniform random variables

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Application to a coupling of a plane wave with an aerial electrical cable (NEMP)

M (1800 s per run): Full-wave simulation (Finite Difference Time Domain)

M_2(0.1 s per run) : Transmission Line theory

Target : 99% quantile (Max induced current)

Application to a coupling of a plane wave with an aerial electrical cable (NEMP)

- Quantile estimation of level 99% of the current:

	1st stratum	2nd stratum	3rd stratum	4th stratum
Quantiles (A) of **TLT** obtained by 10000 realizations from MC simulation:	50 input realizations for which $Z \in [z_0; z_{0.5}]$ = [0.3; 33.5 A]	50 input realizations for which $Z \in [z_{0.5}; z_{0.9}]$ = [33.5; 66.9 A]	50 input realizations for which $Z \in [z_{0.9}; z_{0.99}]$ = [66.9; 115.1 A]	50 input realizations for which $Z \in [z_{0.99}; z_1]$ = [115.1; 181.2 A]

FDTD

Estimation of the 99% quantile: ➡ $\hat{Y}_{CS}(0.99)$=72.0 A

This estimation $\hat{Y}_{CS}(0.99)$ is rather close to the reference empirical quantile given by 1000 realizations from MCS: $\hat{Y}_{EE}(0.99)$=75.8 A

Application to a coupling of a plane wave with an aerial electrical cable (NEMP)

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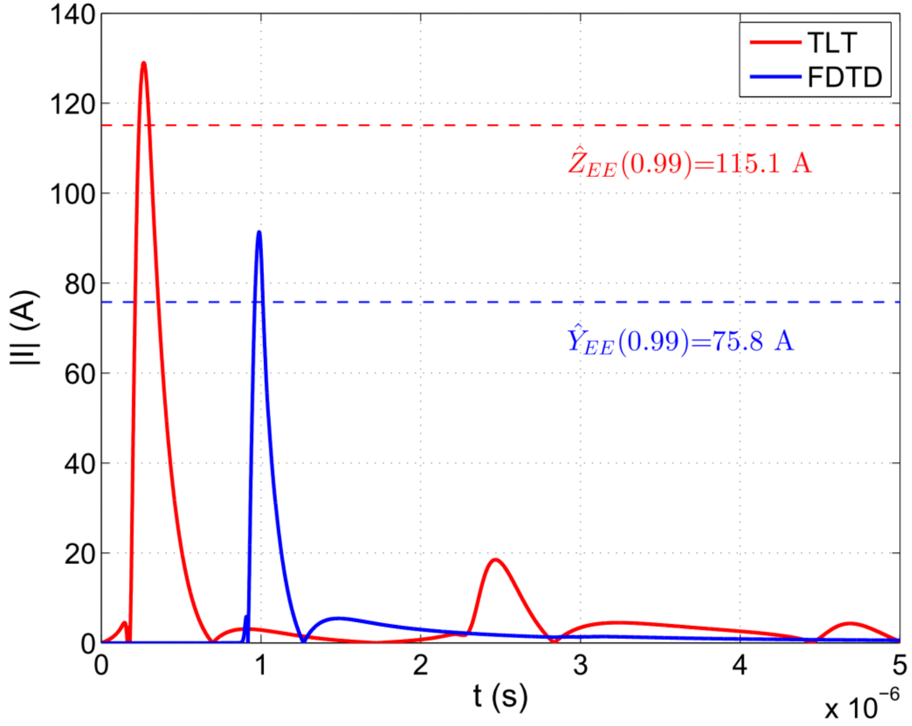
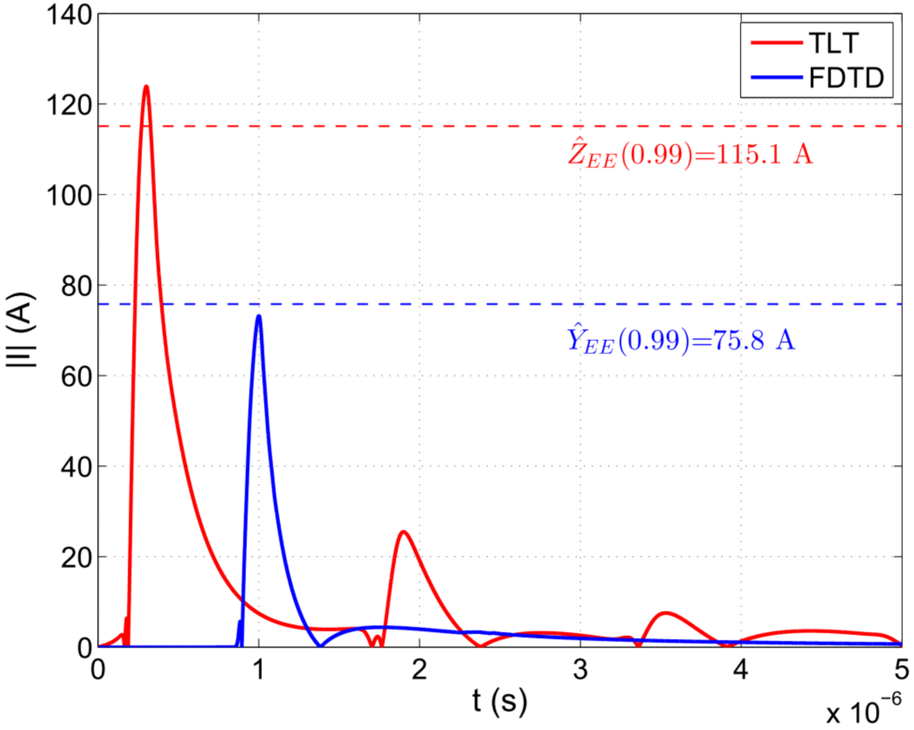
Note that the TLT estimation of the quantile was completely wrong!

Application to a coupling of a plane wave with an aerial electrical cable (NEMP)

- Illustration of the correlation between TLT and FDTD at the tail of their distribution





Correlation between TLT and FDTD

$$\hat{\rho}_I = 0.8$$

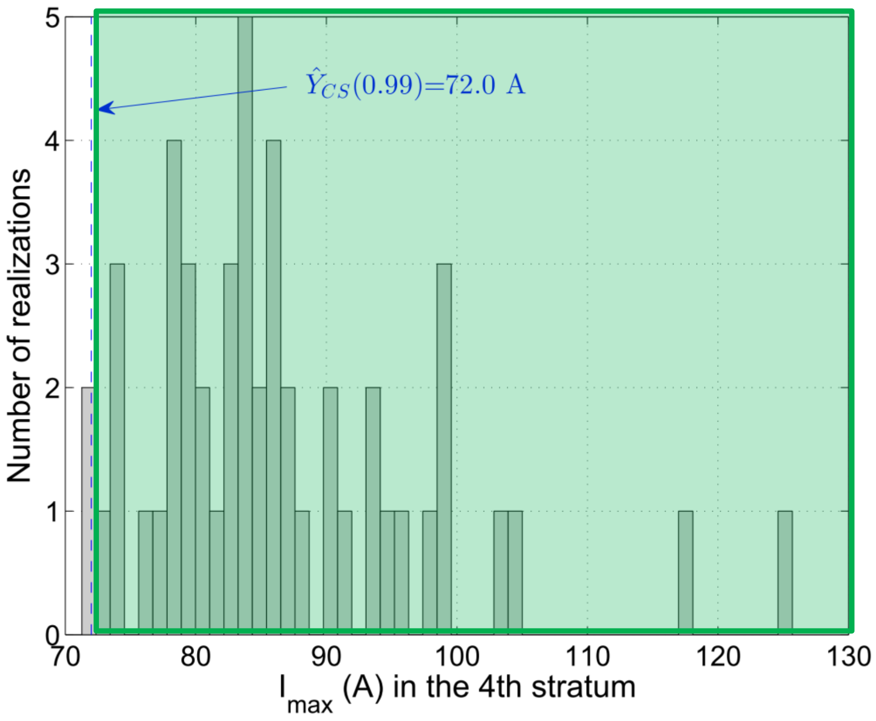
Application to a coupling of a plane wave with an aerial electrical cable (NEMP)

TLT

4^{th} stratum

50 input realizations for which
$Z \in [z_{0.99}; z_1]$
= [115.1; 181.2 A]

FDTD



Among 50 input realizations in FDTD, 48 current values were higher than the CS quantile $\hat{Y}_{CS}(0.99)$=72.0 A

50 computations by CS in the 4th stratum → Max(Imax)=125.7 A

1000 computations from MC simulation → Max(Imax)=104.5 A

→ TLT allowed to identify extreme current values with FDTD

Conclusion

- Controlled stratification is a useful statistical trick: Acts a smart-samping method to explore extreme events

- It is fueled by a « correlated » simple model

- Surrogate models are possible candidates

T. Houret et. al, "Controlled Stratification Based on Kriging Surrogate Model: An Algorithm for Determining Extreme Quantiles in Electromagnetic Compatibility Risk Analysis," in IEEE Access, vol. 8, pp. 3837-3847, 2020

- But correlation of extreme events outputs is enough
  - Approximate models are interesting candidates if correlation is maintained:
    - Physical models
    - Coarse mesh numerical models.

Some References

T. Houret, P. Besnier, S. Vauchamp and P. Pouliguen, "Controlled Stratification Based on Kriging Surrogate Model: An Algorithm for Determining Extreme Quantiles in Electromagnetic Compatibility Risk Analysis," in IEEE Access, vol. 8, pp. 3837-3847, 2020

M. Larbi, P. Besnier, B. Pecqueux, "The Adaptative Controlled Stratification Method Applied to the Determination of Extreme Interference Levels in EMC Modeling with Uncertain Input Variables," in IEEE transactions on electromagnetic compatibility, vol. 58, n°2, pp. 543-552, Apr. 2016

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